Exam F (Part I) **Name**

1. The hour hand of the clock on a clocktower is 2 feet long. The minute hand is $2\frac{1}{2}$ feet long. The clock loses one minute per hour.

i. How many feet does the tip of the hour hand travel in 72 hours?

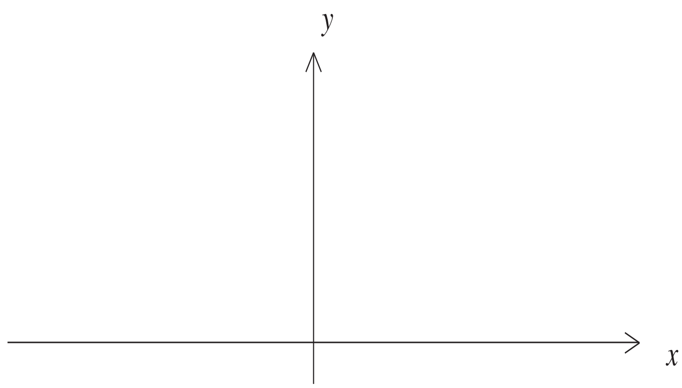
ii. How many feet does the tip of the minute hand travel in 72 hours?

2. Using Leibniz's tangent method compute the slope of the tangent to the curve $y^2 = x^3 + 4$ at $P = (1, \sqrt{5})$. At which points Q does the graph have an horizontal tangent?

3. Three non-negative numbers, at least two of which are equal, have a sum of 30. What is the maximum possible value of the product of the three numbers? Carefully justify the assertion that the value you obtain is the maximum possible value. Is there a minimum possible value of their product? Is there a maximum value of the product if the numbers do not all have to be non-negative?

4. Consider the parabola $y = -x^2 + 12$ and place a rectangle under it so that the base of the rectangle lies on the x-axis and the two upper vertices are on the parabola. Sketch the typical situation below. Determine the largest area that such a rectangle can have.

5. Consider the function $f(x) = 1/x^2$ with $x > 0$, and let (x_0, y_0) be a typical, but fixed point on this graph. Sketch the graph of the function as well as tangent line to the graph at this point.



i. Determine an equation of the tangent line in terms of the variables x and y and the constant x_0.

ii. Compute the area of the triangle given by the tangent line, the x-axis, and the y-axis in terms of x_0.

iii. Discuss the question: For which x_0 is the area of this triangle a maximum and for which x_0 is it a minimum?

6. Consider the function $f(x) = 16 - x^2$ with $-2 \leq x \leq 2$. Select the points

$$-2 < -1.5 < -1 < -0.5 < 0 < 0.5 < 1 < 1.5 < 2$$

on the x-axis between -2 and 2.

i. Compute an approximation of the area under the graph of $f(x)$ over $-2 \leq x \leq 2$ using the approach of Leibniz described in Section 5.6.

ii. Use Archimedes theorem to find the same area.

iii. Use the fundamental theorem of calculus to find this area.

7. Consider the upper half of the circle of radius 1 with center at the origin. For any x with $-1 \leq x \leq 1$, let $G(x)$ be the area under this curve (and above the x-axis) from -1 to x. What is the derivative of the function $G(x)$ equal to?

8. Recall that the power series expansion $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$ converges for all x satisfying $-1 < x < 1$.

i. Compute the power series expansion of $\frac{1}{1-3x^2}$ and determine the set of x for which it converges.

ii. Use this power series to approximate $\displaystyle\int_0^{\frac{1}{2}} \frac{1}{1-3x^2}\,dx$ with two decimal place accuracy.

9. The pair of functions $x(t) = t - 1$ and $y(t) = \frac{1}{2}t^2 - 3$ are the coordinates of a point moving in the xy-coordinate plane as functions of the elapsed time $t \geq 0$.

i. Describe the path along which the point moves.

ii. Express that speed of the point as a function of time.

Formulas and Facts: $v(t) = \sqrt{x'(t)^2 + y'(t)^2}$
Area of circular sector equals: $\frac{1}{2}\theta r^2$
Archimedes's theorem: Area of parabolic section $= \frac{4}{3} \times$ Area of inscribed triangle.
Cavalieri's principle: Consider two regions in the plane and let C and D be their areas. Let c_x and d_x be the respective cross-sections of the two regions for all x on some coordinate axis with $a \le x \le b$. If $c_x = kd_x$ for all x and some constant k, then $C = kD$.

$F_P = \frac{8\kappa^2 m}{L}\frac{1}{r_P^2}$ $\quad F = G\frac{Mm}{r^2}$ $\quad \frac{a^3}{T^2} = \frac{GM}{4\pi^2}$